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What the United States Can Learn From Singapore's World-Class Mathematics System

(and what Singapore can learn from the United States):

An Exploratory Study

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EXECUTIVE SUMMARY

INTRODUCTION

Singaporean students ranked first in the world in mathematics on the Trends in International Mathematics and Science Study-2003; U.S. students ranked 16th out of 46 participating nations at grade 8 (Mullis, et al., 2004). Scores for U.S. students were among the lowest of all industrialized countries. Because it is unreasonable to assume that Singaporean students have mathematical abilities inherently superior to those of U.S. students, there must be something about the system that Singapore has developed to teach mathematics that is better than the system we use in the United States.

This exploratory study compares key features of the Singapore and U.S. mathematics systems in the primary grades, when students need to build a strong mathematics foundation. It identifies major differences between the mathematics frameworks, textbooks, assessments, and teachers in Singapore and the United States. It also presents initial results from four pilot sites that introduced the Singapore mathematics textbook in place of their regular textbooks.

Analysis of these evidentiary streams finds Singaporean students more successful in mathematics than their U.S. counterparts because *Singapore has a world-class mathematics system with quality components aligned to produce students who learn mathematics to mastery.* These components include Singapore's highly logical national mathematics framework, mathematically rich problem-based textbooks, challenging mathematics assessments, and highly qualified mathematics teachers whose pedagogy centers on teaching to mastery. Singapore also provides its mathematically slower students with an alternative framework and special assistance from an expert teacher.

The U.S. mathematics system does not have similar features. It lacks a centrally identified core of mathematical content that provides a focus for the rest of the system. Its traditional textbooks emphasize definitions and formulas, not mathematical understanding; its assessments are not especially challenging; and too many U.S. teachers lack sound mathematics preparation. At-risk students often receive special assistance from a teacher's aide who lacks a college degree. As a result, the United States produces students who have learned only to mechanically apply mathematical procedures to solve routine problems and who are, therefore, not mathematically competitive with students in most other industrialized countries.

The experiences of several of the U.S pilot sites that introduced the Singapore mathematics textbooks without the other aspects of the Singaporean system also illustrate the challenges teachers face when only one piece of the Singapore system is replicated. Some pilot sites coped successfully with these challenges and significantly improved their students' mathematics achievement, but others had great difficulty. Professional training improved the odds of success, as did serving a stable population of students who were reasonably able with mathematics. These mixed results further reinforce the comparative findings that the U.S. will have to consider making comprehensive reforms to its school mathematics system if we are to replicate the Singaporean successes.

The U.S. mathematics system has some features that are an improvement on Singapore's system, notably an emphasis on 21st century thinking skills, such as reasoning and communications, and a focus on applied mathematics. However, if U.S. students are to become successful in these

areas, they must begin with a strong foundation in core mathematics concepts and skills, which, by international standards, they presently lack.

EXPLORATORY METHODOLOGY

Carrying out in-depth analyses on systems as different as those in Singapore and the United States poses serious methodological challenges. Singapore has a centralized mathematics system, with detailed and consistent implementation procedures. This makes analysis of the separate components of their system relatively straightforward. Characterizing the decentralized U.S. mathematics system, in contrast, is difficult. We elected to rigorously study the components of the U.S. system by selecting typical examples from the wide variety available in each component area:

- **Standards:** The United States has no national standards, but many states' standards use the National Council of Teachers of Mathematics (NCTM) framework as a model. We used the NCTM standards in our analyses as a proxy for states that use a grade-band (e.g., K–2, 3–5), rather than a grade-by-grade structure in their standards. However, because many states are currently shifting to a grade-by-grade structure in response to NCLB, we supplemented our analysis by also examining standards from seven states (Exhibit A) that organize content grade by grade. These states are home to approximately one-third of all U.S. students.

- **Textbooks:** We limited our analysis to one traditional and one nontraditional U.S. mathematics textbook.

- **Assessments:** We used sample assessment items from the federally supported National Assessment of Educational Progress (NAEP) and from assessments from the same seven states whose standards we examined in our comparative analysis.

- **Teachers:** For analyses of teacher quality in the United States, we drew from national surveys on teacher education and from teacher preparation standards. We also examined sample problems from teacher licensing exams.

Exhibit A. The Average Number of Topics per Grade in Selected U.S. State Mathematics Frameworks Compared With Singapore's

	Avg. No. of Topics per Grade	Ratio to Sing.
Singapore	15	—
California	20	1.3
Florida	39	2.6
Maryland	29	1.9
New Jersey	28	1.9
N. Carolina	18	1.2
Ohio	26	1.7
Texas	19	1.3

In evaluating the results from the four Singapore textbook pilot sites – Baltimore, Maryland; Montgomery Country, Maryland; North Middlesex, Massachusetts; and Paterson, New Jersey – we relied on data previously collected by the districts rather than on uniform data collected specifically

for this study. Because different sites used different sites assessments, usually the state assessment, results are not completely comparable. The reader should be mindful of study limitations in all areas of comparison.

PREFERRED FEATURES OF THE SINGAPORE MATHEMATICS SYSTEMS

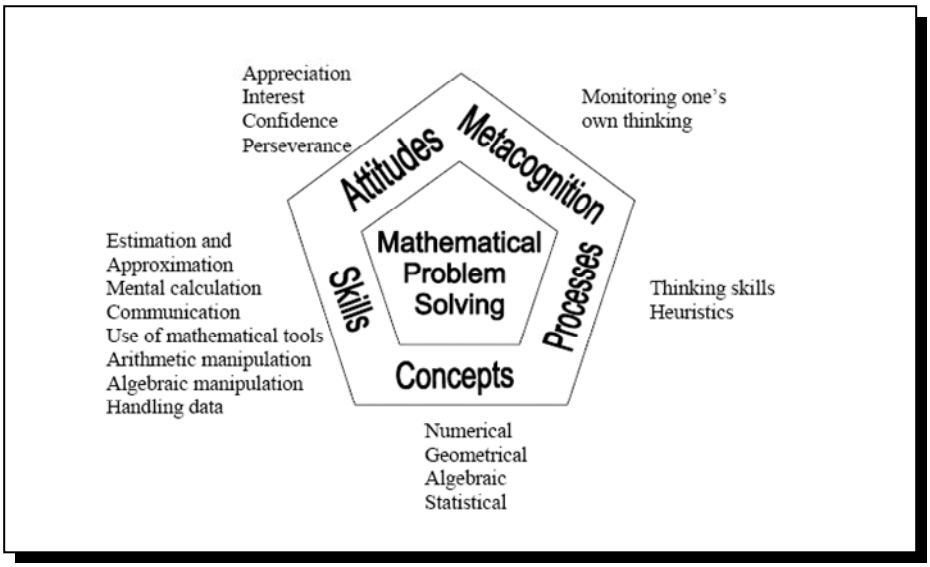
Our key findings show the advantages conferred by components of Singapore's mathematics system in comparison to similar components in the U.S. system.

Frameworks

A mathematically logical, uniform national framework that develops topics in-depth at each grade guides Singapore's mathematics system. The U.S. system, in contrast, has no official national framework. State frameworks differ greatly; some resemble Singapore's, whereas others lack Singapore's content focus.

Singapore's framework, shown in Exhibit B, lays out a balanced set of mathematical priorities centered on problem solving. It includes an emphasis on computational skills along with more conceptual and strategic thinking processes. The framework covers a relatively small number of topics in-depth and carefully sequenced grade-by-grade, following a spiral organization in which topics presented at one grade are covered in later grades, but only at a more advanced level. Students are expected to have mastered prior content, not repeat it.

Exhibit B. Singapore's Mathematics Framework



The NCTM framework, while emphasizing higher order, 21st century skills in a visionary way, lacks the logical mathematical structure of Singapore's framework. It identifies content only within broad grade bands (e.g., K–2, 3–5) and only in general terms, thus providing inadequate content guidance to educators.

The seven state frameworks we examined exhibit varying degrees of focus, although none is as focused as Singapore's. Exhibit A shows that three of the states, California, North Carolina, and

Texas, have frameworks that are similar to Singapore's, within 30 percent, in the average number of topics covered per grade. Two of these states, North Carolina and Texas, were praised in the 1990s as states where education reform had been particularly successful. Both states' NAEP mathematics scores improved significantly. The similarity between these states and Singapore suggests a correlation between focused frameworks and good test performance.

By contrast, the frameworks of Florida, Maryland, New Jersey, and Ohio exceeded Singapore's average numbers of topics per grade by 70 to 160 percent. If Singapore's excellent test performance is evidence that its curriculum exposes students to about the right number of topics per grade, then these states' test performance suggests they cover too many topics and should reduce breadth of coverage and deepen topic instruction.

Singapore recognizes that some students may have more difficulty in mathematics and provides them with an alternative framework; the U.S. frameworks make no such provisions.

Singapore's alternative mathematics framework for lower performing students covers all the mathematics topics in the regular framework, but at a slower pace and with greater repetition. Singapore also provides its slower students with extra help from well-trained teachers. NCTM and the states we examined provide no alternative framework for slower mathematics students. Moreover, such students are often unofficially tracked into slower mathematics courses, but unlike in Singapore, these students are seldom taught all the required mathematics material. Evaluations have shown that they frequently receive their extra help from teacher's aides who lack college degrees.

Textbooks

Singapore's textbooks build deep understanding of mathematical concepts through multistep problems and concrete illustrations that demonstrate how abstract mathematical concepts are used to solve problems from different perspectives. Traditional U.S. textbooks rarely get beyond definitions and formulas, developing only students' mechanical ability to apply mathematical concepts.

There is a clear difference in how Singapore and traditional U.S. textbooks develop mathematical concepts. The Singapore texts are rich with problem-based development in contrast to traditional U.S. texts that rarely get much beyond exposing students to the mechanics of mathematics and emphasizing the application of definitions and formulas to routine problems. While such books are filled with real-world illustrations, these seem to serve mainly to show students that mathematics concepts have real-world representations. The illustrations make virtually no contribution to helping students understand how to use the mathematics to solve real-world problems.

The Singapore illustrations also feature a concrete to pictorial to abstract approach. Many students who have difficulty grasping abstract mathematical concepts would benefit from visual representations of mathematical ideas. As part of this approach, the Singapore illustrations demonstrate how to graphically decompose, represent, and solve complicated multistep problems.

Another hindrance to the development of U.S. students' mathematical understanding is the U.S. texts' lack of focus. Singapore's textbooks follow its mathematically logical national framework, but U.S. textbooks must serve multiple state markets. To do so, they find it necessary to cover almost twice as many topics per grade so that all topics from many states' frameworks can be covered. Consequently, individual topic coverage in U.S. textbooks is much shorter and less

comprehensive than what is found in Singaporean texts. In fact, Singapore students are expected to complete about one thorough lesson focused on a single topic per week, while U.S. students are expected to complete about one lesson on a narrowly focused topic each day.

Finally, both Singaporean textbooks and U.S. textbooks "spiral" mathematical content – returning in successive years to the same concepts. However, while the spiral in U.S. textbooks includes significant repetition and reteaching of the same content in two or three consecutive grades, the Singapore textbooks assume that what was previously taught was learned. In other words, Singapore textbooks do not repeat earlier-taught content, because students are taught to mastery the first time around.

Assessments

The questions on Singapore's high-stakes grade 6 Primary School Leaving Examination (PSLE) are more challenging than the released items on the U.S. grade 8 National Assessment of Education Progress (NAEP) and the items on the grade 8 state assessments.

Singapore's grade 6 assessment contains almost double the percentage of constructed-response items as the U.S. grade 8 NAEP and a much higher proportion than that of state assessments. This is an important difference because constructed-response questions generally are more suitable for demonstrating students' higher-level cognitive process in mathematics.

Overall, Singapore's grade 6 assessment also contains a much greater percentage of items that could be characterized as more difficult than either the U.S. 8th grade NAEP or any of the state assessments we examined. These differences are in part the result of NAEP's policy of not including items with very high (or very low) p-values. Many PSLE problems require using multiple steps, solving for an intermediate unknown, or using a nonroutine solution that goes beyond a simple application of a definition or formula. Singapore's most challenging questions are designed to help Singapore identify the best students. These are more difficult than the most challenging questions on the state grade 8 assessments as well as on NAEP.

As a way to hold schools as well as students accountable for performance, Singapore uses a measure of each school's *value-added* contribution to student achievement. The U.S. Adequate Yearly Progress (AYP) measure in No Child Left Behind does not.

A value-added measure of school performance looks at the growth in student outcomes after adjusting for the initial performance. Singapore aggregates individual student test results on its national grades 6 and 10 exams by school. It then compares the expected growth in school outcomes, adjusted for a school's students' initial grade 6 performance, with actual growth to obtain a value-added indicator of a school's performance. Schools that perform above expectations are recognized and rewarded.

The U.S. requirements for AYP under NCLB hold each school accountable for annual growth toward the goal of having all students reach proficiency on state assessments. NCLB allows students to leave schools that have a record of poor performance and are in need of improvement and move to schools with high-performing students. However, a high-performing school that takes students who are low performers is penalized because it will have to make greater gains to meet AYP targets. Schools in this situation may be discouraged from taking low-performing students; Singapore's value-added measure of school progress removes this disincentive.

Teachers

Singaporean elementary school teachers are required to demonstrate mathematics skills superior to those of their U.S. counterparts before they begin teacher training. At every phase of pre- and post-service training, they receive better instruction both in mathematics content and in mathematics pedagogy.

Singapore's teachers must take a stringent examination before being accepted to education school, and while they are students, they are paid a teacher's salary. By contrast, the SAT mathematics scores of entering U.S. elementary education majors are among the lowest of all college students.

After content-driven pre-service preparation, Singaporean teachers are encouraged to continue to improve their knowledge and skills through 100 hours of required annual professional training. U.S. education majors, in contrast, take fewer formal mathematics courses than the average college graduate. The major U.S. teacher screening and licensing exams, the PRAXIS I and II, consist only of multiple-choice questions that, based on released items, appear far easier than items from the exam that Singapore gives to 6th graders. An alternative version of the PRAXIS II (10140) poses more challenging mathematics problems, consistent with having teachers demonstrate higher-order thinking skills, but currently no state requires prospective elementary teachers to pass this more difficult test. After entering the profession, U.S. elementary school teachers typically spend only about a quarter of the 100 hours per year that their Singaporean counterparts spend on professional development activities. The most common form of professional development in the United States is the short-term workshop, widely admitted to be ineffective for changing practice.

AREAS OF STRENGTHS IN THE U.S. MATHEMATICS SYSTEM COMPARED WITH SINGAPORE'S SYSTEM

Although the U.S. mathematics program is weaker than Singapore's in most respects, the U.S. system is stronger than Singapore's in some areas.

The U.S. frameworks give greater emphasis than Singapore's framework does to developing important 21st century mathematical skills such as representation, reasoning, making connections, and communication.

However, to develop these skills in students, the U.S. frameworks need to do a better job of integrating them with rigorous mathematics content.

The U.S. places a greater emphasis on applied mathematics, including statistics, probability, and real-world problem analysis.

The U.S. mathematics frameworks stress data analysis and probability, whereas the Singapore framework treats statistics in a strictly theoretical way. *Everyday Mathematics*, the nontraditional textbook we examined, uses a problem-based learning approach, which presents multistep real-world mathematics problems. Such applications give students practice in understanding how to apply mathematics in practical ways. However, the *Everyday Mathematics* lessons use real-world applications without providing the foundation of the strong conceptual topic development found in Singapore's textbooks. Even though Singapore's textbooks would benefit from

more real-world applications, their emphasis on conceptual development of mathematics and problem-based learning make them superior to U.S. textbooks overall.

PILOT SITE FINDINGS: MIXED RESULTS

The two pilot sites (out of four) that had both a stable population of higher performing students and a clear staff commitment to support the introduction of the Singapore mathematics textbooks produced sizeable improvements in student outcomes.

In North Middlesex, Massachusetts, the school system of about 5,000 was selected by the state education agency to pilot the Singapore textbooks. Over two years, the percentage of those students who participated in the Singapore pilot and scored at the advanced level on the grade 4 Massachusetts assessment increased by 32 percent over two years. The pilot schools had strong district and staff support. Over two years, Baltimore's Ingenuity Project increased the proportion of its students who scored at the 97th percentile or above by 17 percent. The Ingenuity Project serves gifted Baltimore students and can select highly skilled teachers capable of teaching the mathematical reasoning underlying the challenging Singapore problems.

The two other Singapore pilot sites, which in one case had uneven staff commitment to the project and in the other case had a more transient, lower income population, produced uneven or disappointing results.

- The Montgomery County outcomes were positively correlated with the amount of *professional training* the staff received. Two Singapore pilot schools availed themselves of extensive professional development and outperformed the controls; two other pilot schools had low staff commitment coupled with low exposure to professional training and were actually outperformed by the controls. Professional training is important in helping teachers understand and explain the nonroutine, multistep problems in the Singapore textbooks. Teachers also need preparation to explain solutions to Singapore problems, which often require students to draw on previously taught mathematics topics, which the Singapore textbook, in contrast to U.S. textbooks, does not reteach.

- The Paterson, New Jersey, school, with an *annual student turnover of about 40 percent,* fared no better on the New Jersey grade 4 test than the district average over two years. Having such a high student turnover meant that many 4th graders were exposed to the Singapore mathematics textbook for the first time - by definition, not a fair test of the cumulative effects of exposure to the textbook.

Several sites also had difficulties because the Singapore textbooks did not match their state's mathematics priorities.

The most serious mismatch occurred in Paterson, where grade 4 teachers supplemented the Singapore mathematics textbook with their U.S. textbook to cover a few topics, notably statistics and probability, that were on their grade 4 state assessment but not in the Singapore grade 4 textbook. Unfamiliar with the pedagogy laid out in Singaporean Teachers' Guides, several sites were also concerned that the Singapore textbooks did not stress written communication skills by requiring students to explain their answers.

The challenges in using the Singaporean textbooks, such as the lack of teacher preparation, the discrepancies between the topics on the state assessments and the topics in the textbooks at particular grades, and the lack of prior student exposure to the Singapore curriculum, are not challenges faced in Singapore where mathematics textbooks and teacher preparation are aligned to the content in the common framework and where students are held accountable for learning all topics to mastery as they go along.

CONCLUSION

Reform Options

Each component of Singapore's educational system is designed to enhance the mathematical proficiency of students and their teachers. If the United States is to reform its mathematics system so that it more closely resembles Singapore's successful system, the country needs to consider several options for improving each of the components of the system. The options are organized by how much change from current practice would be required and, hence, by how difficult it would be to gain political acceptance for them.

Tinkering Options: Improve or extend existing reforms. States could revise their frameworks to better match Singapore's content grade by grade and strengthen implementation of NCLB reforms for highly qualified teachers to ensure that teachers who meet the NCLB standards actually demonstrate that they understand mathematics content and how to teach it. The federal government could work with the states to produced a national bank of mathematics test items to encourage greater comparability across the states.

Leveraging Options: Use market leverage to bring about improvement. Professional organizations could develop an independent and objective textbook rating system that assesses the depth of mathematics content in textbooks, much as the American Association for the Advancement of Science has already piloted in the sciences.

Program Strengthening Options: Stay within the current U.S. education structure but substantially strengthen the mathematical depth and rigor of the current components of the U.S. mathematical system. U.S. textbooks could be reorganized so that they closely conform to the logical topic organization, rich problem-based approach, and varied pictorial representations of mathematical concepts found in Singaporean texts. Eighth-grade student assessments and teacher-licensing exams could be strengthened so that, at a minimum, they are at least as challenging as Singapore's grade 6 student assessment.

Systemic Reform Options: Strengthen features of the U.S. mathematics system so that it more closely resembles Singapore's integrated, national mathematics system. Such steps might include introducing a national mathematics framework, a national mathematics assessment, and value-added accountability measures of school performance.

Further Validation of Exploratory Findings

Our exploratory results have identified key differences between the U.S. and Singapore mathematics systems. These differences suggest potentially significant reforms that could improve the U.S. mathematics system, but these findings require further validation from larger, more

scientific studies. The suggested reforms need more thorough analyses and, ideally, small-scale introduction prior to going to scale. Only through such further study can we build on our exploratory findings to assess whether adopting the features that have produced a quality mathematics system for Singapore would significantly improve the performance of the U.S. mathematics system and better meet the challenging performance goals set by NCLB.